



Solving Neutrosophic Bi-objective Assignment Problem using Transformation Techniques

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Abstract

The assignment problem is the core concept of fundamental optimization problem in the field of engineering and management sciences. In some situations, decision maker needs to optimize the assignment cost, assignment time, deterioration rate, profit and so on simultaneously which gives way to multi-objective assignment problem. In practical problems, there are some unpredictable conditions due to time constraints, limitations in data, inaccuracy in measurements and so on. So, the single valued trapezoidal neutrosophic numbers is considered here to handle this fact. The objective of this paper is to determine the optimal compromise solution for the neutrosophic bi-objective assignment problem by considering all the parameters as single valued trapezoidal neutrosophic number. Initially using score function, we convert the neutrosophic problem into its deterministic problem. Transformation techniques can assist the decision makers to present their neutral views and manage uncertainties efficiently in the decision-making situations. So, we have developed the transformation techniques for assignment problem such as linear, hyperbolic, exponential membership function, fuzzy programming approach, neutrosophic compromise programming approach, global criteria method and weighted sum method to transform the deterministic bi-objective assignment problem into its single objective assignment problem under neutrosophic environment. The reduced problem is then solved by LINGO to find the optimal compromise solution and it lets the decision maker to specify the targets. To evaluate the viability and accuracy of our study, numerical illustrations have been conducted and comparisons have been made. Sensitivity analysis have been performed in this study. Finally, conclusions and future explorations are depicted.

Keywords: single-valued trapezoidal neutrosophic numbers; score function; fuzzy programming approach; neutrosophic compromise programming approach; global criteria method; weighted sum method; membership functions.

1 Introduction

The assignment problem (AP) aims to minimize or maximize the objective function by assigning different manufacturers to distinct types of medical devices. In some situations, the decision maker (DM) needs to minimize the assignment cost, assignment time, deterioration rate and so on. In order to deal this situation, we include AP with multi-objective functions. In recent decades, numerous articles are published in the field of multi-objective assignment problem (MOAP). Zadeh [32] introduced global criteria method, Zimmermann [34] introduced fuzzy programming approach and Rao [20] introduced weighted sum method to reduce the multi-objective problem (MOP) into the single objective problem (SOP) respectively. Kagade and Bajaj [13] developed the fuzzy programming approach (FPA) using various membership function (MF) to solve MOAP. Tilva and Dhodiya [28] established an approach using exponential MF for solving MOAP. Frau and Pilotta [10] proposed the coordinate search method for solving the optimization problems. Ishak and Marjugi [12] modified the conjugate gradient method to solve the optimization problems.

Decision makers encounter inadequate and inaccurate data in numerous disciplines including research, engineering and management. Depending on the circumstances, fuzzy numbers (FN), generalized FN, intuitionistic FN, interval valued FN, neutrosophic numbers and so on are employed to handle these situations. Zadeh [33] introduced a fuzzy set that is based on its membership functions to manage imprecise problems. [8] applied linear, exponential, logarithmic, hyperbolic and quadratic MF to obtain optimal compromise solution (OCS) for solving fuzzy multi-objective problem in decision making. Kumar and Gupta [17] proposed the new method for solving fuzzy AP with different MF using Yager ranking approach. Pramanik and Biswas [19] utilized fuzzy goal programming approach based on priority to analyze the MOAP under uncertain parameters. Ahmad and Adhami [1] proposed the neutrosophic compromise programming approach (NCPA) to reduce the fuzzy multi-objective transportation problem (MOTP) into its fuzzy single objective TP. Vinoliah and Ganesan [29] proposed the weighted average method for solving the MOAP under fuzzy environment. Khalifa [16] proposed a new approach namely, optimal flowing method for solving fuzzy MOAP to find the ideal and efficient solutions.

Atanassov [3] postulated the intuitionistic fuzzy (IF) that combines both MF and non-MF. Singh and Yadav [24] tackled the intuitionistic fuzzy multi-objective LPP with mixed constraints using various MF under intuitionistic environment. Mahajan and Gupta [18] employed linear, exponential and hyperbolic MF to solve intuitionistic fuzzy MOTP (IFMOTP). El Sayed and Abo-Sinna [9] introduced a technique using linear, hyperbolic and parabolic MF for solving fractional IFMOTP. Ahmadini and Ahmad [2] proposed an approach using linear, exponential and hyperbolic MF for IFMOLPP under neutrosophic environment. Singh et al. [25] established a novel approach using various MF to overcome the intuitionistic fuzzy bi-level multi-objective problem.

In practical problems, there are some hesitations due to time constraints, limitation in data, inaccuracy in measurements and so on. Based on this reality, decision maker considers indeterminacy when solving optimization problems. Neutrosophic set (NS) is highly significant and well suited to handle such broad structure of problems in order to overcome such reluctance in both fuzzy and IF. Smarandache [26] introduced the structure of NS for solving the real-world problem in a reasonable manner. Single-valued neutrosophic (SVN) sets are useful in solving many real-world problems. Wang et al. [30] established various properties and operations of SVN set, which requires a crucial role in NS theory. Deli and Subas [7] proposed the weighted aggregation operator for solving multicriteria decision-making problems based on single-valued trapezoidal neutrosophic numbers (SVTrNNs). Bera and Mahapatra [4] proposed a solution approach for solving AP under a neutrosophic environment. Khalifa [14] proposed the weighted tchebycheff

method for solving MOAP under neutrosophic numbers. Khalifa and Kumar [15] modified the weighted tchebycheff technique for interval-valued trapezoidal neutrosophic AP. Wang et al. [31] proposed a method using linear and parabolic MF for solving multi-objective LPP under triangular neutrosophic numbers. Harnpornchai and Wonggattaleekam [11] proposed a solution approach to solve the AP based on neutrosophic set. Sandhiya and Anuradha [22] proposed fixing point approach for solving the bi-objective AP under neutrosophic environment. Sandhiya and Anuradha [23] proposed neutrosophic FPA for solving the neutrosophic MOAP. Buvareshwari and Anuradha [6] proposed left-width for solving the interval-valued neutrosophic MOAP. Srinivas and Prabakaran [27] proposed a progressive strategy for solving neutrosophic MOAP.

Sensitivity analysis (SA) is used to examine the impact of variations in the coefficients of objective functions and right hand side constraints on the optimal value of the objective functions as well as to determine the validity ranges. The main purpose of SA is to identify the sensitivity ranges without affecting the current optimal solution, which is a special estimation to minimize the risk of erroneous solutions. Samanta et al. [21] performed the SA for the objective function parameters. Bind et al. [5] performed the SA for the objective function parameters. Table 1 lists the summary of the literature survey and the present paper.

From Table 1, we have noted the gaps and contribution which are presented below:

- Researchers such as Zadeh [32] introduced the weighted sum method to reduce the MOLPP into LPP. Zimmermann [34] introduced the fuzzy programming approach to reduce the MOLPP into LPP. Rao [20] introduced the global criteria method to reduce the fuzzy MOTP into fuzzy TP. Ahmad and Adhami [1] introduced the neutrosophic compromise programming approach to reduce the fuzzy MOTP into fuzzy TP. Ahmadini and Ahmad [2] introduced the linear, hyperbolic and exponential MF to reduce the intuitionistic MOLPP into intuitionistic LPP.
- It is clear that the work has been conducted on MOLPP and MOTP using transformation techniques for triangular fuzzy numbers and trapezoidal intuitionistic fuzzy numbers. To the best of our knowledge, MOAP under neutrosophic environment for SVTrNNs using these transformation techniques are very rare in the literature.
- Taking all these gaps into consideration, this motivates us to develop the transformation techniques to transform the multi-objective problem under neutrosophic environment into single objective problem under neutrosophic environment which is a novelty of this study.
- In this paper, we have considered the bi-objective assignment problem (BOAP) under neutrosophic environment. To apply these transformation techniques, the first and the second objective signify the assignment cost and time under neutrosophic environment by considering the parameters as SVTrNNs.
- First, the score function of SVTrNNs [7] is used to reduce the NBOAP into its deterministic BOAP. The reduced BOAP is converted into its single objective AP using the transformation techniques such as linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM.
- The reduced problem is then solved by LINGO to find the optimal compromise solution. Finally, we perform the sensitivity analysis for the objective functions of BOAP.
- These transformation techniques are particularly applicable in the fields of finance and engineering especially when solving complex and uncertain optimization problems. The DM has the flexibility to choose and apply transformation techniques based on the problem needs and priorities.

Table 1: Summary of the literature survey and the present paper.

References	Nature of parameters				Objective		SA	Methods
	Cr	F	IF	NF	Single	Multi		
Kagade and Bajaj [13]	✓					✓		Fuzzy programming approach
Tilva and Dhodiya [28]	✓					✓		Exponential MF
Dhingra and Moskowitz [8]	✓				✓			Linear, exponential, logarithmic, hyperbolic and quadratic MF
Kumar and Gupta [17]	✓				✓	✓		Different MF
Pramanik and Biswas [19]		✓				✓		Fuzzy goal programming approach
Ahmad and Adhami [1]				✓		✓		Neutrosophic compromise programming approach
Vinoliah and Ganesan [29]		✓				✓		Weighted average method
Khalifa [16]		✓				✓		Optimal flowing method
Singh and Yadav [24]		✓				✓		Various MF
Mahajan and Gupta [18]		✓				✓		Linear, exponential and hyperbolic MF
El Sayed and Abo-Sinna [9]		✓				✓		Linear, hyperbolic and parabolic MF
Ahmadini and Ahmad [2]		✓				✓		Linear, exponential and hyperbolic MF
Singh et al. [25]		✓				✓		Linear, parabolic and hyperbolic MF
Bera and Mahapatra [4]		✓				✓		New approach
Khalifa [14]		✓				✓		Weighted Tchebycheff method
Khalifa and Kumar [15]		✓				✓		Weighted Tchebycheff method
Wang et al. [31]		✓				✓		Linear and parabolic MF
Harnpornchai and Wonggattaleekam [11]		✓				✓		New approach
Present paper				✓		✓	✓	GCM, WSM, FPA, NCPA, linear, hyperbolic and parabolic MF

Note: Cr: Crisp, F: Fuzzy, IF: Intuitionistic fuzzy, NF: Neutrosophic fuzzy.

The article is classified as Section 2 follows with preliminaries and essential definitions of neutrosophic sets. The formulation of neutrosophic BOAP is dealt in Section 3. The various transformation techniques is described in Section 4 while Section 5 describes the solution approach. Section 6 depicts a numerical illustration while Section 7 discusses the sensitivity analysis. Section 8 deals with conclusions and future scopes.

2 Preliminaries

In this section, definitions of NS [26], SVN sets [30], SVTrNNs, arithmetic operations and score function of SVTrNNs [7] have been discussed.

Definition 2.1 (Neutrosophic set [26]). Let X be a universe discourse. A NS L in X is characterized by a truth MF $A_{\bar{L}N}(x)$, indeterminacy MF $B_{\bar{L}N}(x)$ and a falsity MF $C_{\bar{L}N}(x)$. $A_{\bar{L}N}(x)$, $B_{\bar{L}N}(x)$ and $C_{\bar{L}N}(x)$ are real standard elements of $[0, 1]$. It can be written as,

$$\bar{L}^N = \{ \langle x, A_{\bar{L}N}(x), B_{\bar{L}N}(x), C_{\bar{L}N}(x) \rangle : x \in X, A_{\bar{L}N}(x), B_{\bar{L}N}(x), C_{\bar{L}N}(x) \in]0^-, 1^+ [\}.$$

There is no restriction on the sum of $A_{\bar{L}N}(x)$, $B_{\bar{L}N}(x)$ and $C_{\bar{L}N}(x)$, so,

$$0^- \leq A_{\bar{L}N}(x) + B_{\bar{L}N}(x) + C_{\bar{L}N}(x) \leq 3^+.$$

Definition 2.2 (Single-valued neutrosophic sets [30]). A SVN $\bar{L}^{SVN}$ of a non-empty set X is defined as follows,

$$\bar{L}^{SVN} = \{ \langle x, A_{\bar{L}N}(x), B_{\bar{L}N}(x), C_{\bar{L}N}(x) \rangle : x \in X \},$$

where $A_{\bar{L}N}(x), B_{\bar{L}N}(x), C_{\bar{L}N}(x) \in [0, 1]$ for each $x \in X$ and $0 \leq A_{\bar{L}N}(x) + B_{\bar{L}N}(x) + C_{\bar{L}N}(x) \leq 3$.

Definition 2.3 (Single-valued trapezoidal neutrosophic number [7]). Let $\sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \in [0, 1]$ and $p, q, r, s \in \mathbb{R}$ such that $p \leq q \leq r \leq s$. Then a SVTrNN, $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ is a special NS on $\mathbb{R}$, whose truth membership, indeterminacy membership and falsity membership functions are given below,

$$\mu_{\tilde{d}^N} = \begin{cases} \sigma_{\tilde{d}^N} \left(\frac{x-p}{q-p} \right), & p \leq x < q, \\ \sigma_{\tilde{d}^N}, & q \leq x \leq r, \\ \sigma_{\tilde{d}^N} \left(\frac{s-x}{s-r} \right), & r \leq x \leq s, \\ 0, & \text{otherwise,} \end{cases}$$

$$\delta_{\tilde{d}^N} = \begin{cases} \frac{q-x+\lambda_{\tilde{d}^N}(x-p)}{l-k}, & p \leq x < q, \\ \lambda_{\tilde{d}^N}, & q \leq x \leq r, \\ \frac{x-r+\lambda_{\tilde{d}^N}(s-x)}{s-r}, & r \leq x \leq s, \\ 0, & \text{otherwise,} \end{cases}$$

$$\rho_{\tilde{d}^N} = \begin{cases} \frac{q-x+\tau_{\tilde{d}^N}(x-p)}{q-p}, & p \leq x < q, \\ \tau_{\tilde{d}^N}, & q \leq x \leq r, \\ \frac{x-r+\tau_{\tilde{d}^N}(s-x)}{s-r}, & r \leq x \leq s, \\ 1, & \text{otherwise,} \end{cases}$$

where $\sigma_{\tilde{d}^N}$, $\lambda_{\tilde{d}^N}$ and $\tau_{\tilde{d}^N}$ denote the maximum truth, minimum indeterminacy, and minimum falsity membership degrees respectively. A SVTrNN $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ may be expressed as an ill-defined quantity of p , which is approximately equal to $[q, r]$.

Definition 2.4 (Arithmetic Operations on SVTrNNs [7]). Let $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ and $\tilde{g}^N = \langle (p', q', r', s'); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle$ be two SVTrNNs. The arithmetic operations on $\tilde{d}^N$ and $\tilde{g}^N$ are

1. $\tilde{d}^N + \tilde{g}^N = \langle (p + p', q + q', r + r', s + s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle$.
2. $\tilde{d}^N - \tilde{g}^N = \langle (p - p', q - q', r - r', s - s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle$.
3. $\tilde{d}^N * \tilde{g}^N = \begin{cases} \langle (pp', qq', rr', ss'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s > 0, \quad s' > 0, \\ \langle (ps', qr', qr', ps'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s < 0, \quad s' > 0, \\ \langle (ss', qq', rr', pp'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s < 0, \quad s' < 0. \end{cases}$
4. $\tilde{d}^N / \tilde{g}^N = \begin{cases} \langle (p/s', q/r', r/q', s/p'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s > 0, \quad s' > 0, \\ \langle (s/s', r/r', q/q', p/p'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s < 0, \quad s' > 0, \\ \langle (s/p', r/q', q/r', p/s'); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \rangle, & s < 0, \quad s' < 0. \end{cases}$
5. $d\tilde{g}^N = h(x) = \begin{cases} \langle (dp, dq, dr, ds); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, & d > 0, \\ \langle (ds, dr, dq, dp); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, & d < 0. \end{cases}$
6. $\tilde{g}^{N^{-1}} = \langle (1/s', 1/r', 1/q', 1/p'); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, \tilde{g}^N \neq 0$.

Definition 2.5 (Score function of SVTrNNs [7]). Let $\tilde{d}^N = \langle (p, q, r, s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ be a single-valued trapezoidal neutrosophic number then the score function of $\tilde{d}^N$ is

$$S(\tilde{d}^N) = \frac{1}{16} [p + q + r + s] (2 + \sigma_{\tilde{d}^N} - \lambda_{\tilde{d}^N} - \tau_{\tilde{d}^N}).$$

Definition 2.6 (Efficient solution [34]). A point $\hat{y}$ is said to be an efficient solution of MOAP if there does not exist any y in the feasible region such that $Z^m(y) \leq Z^m(\hat{y})$ for all $m = 1, 2, \dots, M$ and $Z^m(y) < Z^m(\hat{y})$ for at least one m .

In this article, the assignment problem with two conflicting objective functions under neutrosophic environment is formulated. The neutrosophic bi-objective assignment problem (NBOAP) is transformed into its deterministic BOAP using score function. The deterministic BOAP is converted into its single objective AP using various transformation techniques and then it is solved by LINGO to obtain OCS. The formulation of neutrosophic BOAP is elaborated in the next section.

3 Formulation of Neutrosophic Bi-objective Assignment Problem (NBOAP)

Let us assume that there are n medical devices in a manufacturing company and the company has n manufacturers processing the medical devices. Each medical device in a manufacturing company must be associated with only one manufacturer. Our goal is to minimize or maximize the

objective function by assigning manufacturers to medical devices. In reality, the parameters such as assignment cost and assignment time are imprecise due to some unpredictable reasons such as opinion by the doctors and fluctuations in market. In these situations, the uncertain parameters arise in the structure of SVTrNNs which provides greater flexibility to the DM. The mathematical formulation of NBOAP is

$$(P_1) \text{ Minimize } \tilde{Z}^{(1)N}(x) = \sum_{i=1}^n \sum_{j=1}^n \tilde{c}_{ij}^N \tilde{x}_{ij}^N, \quad (1)$$

$$\text{Minimize } \tilde{Z}^{(2)N}(x) = \sum_{i=1}^n \sum_{j=1}^n \tilde{t}_{ij}^N \tilde{x}_{ij}^N. \quad (2)$$

Subject to:

$$\sum_{j=1}^n \tilde{x}_{ij}^N = 1^N (\text{only } i^{th} \text{ resource would be assigned to the } j^{th} \text{ activity}), \quad (3)$$

$$\sum_{i=1}^n \tilde{x}_{ij}^N = 1^N (\text{only } j^{th} \text{ activity would be assigned to the } i^{th} \text{ resource}), \quad (4)$$

$$\tilde{x}_{ij}^N = 0^N \text{ or } 1^N, \text{ for all } i \text{ and } j, \quad (5)$$

where

$\tilde{c}_{ij}^N = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4; c_{ij}^{'1}, c_{ij}^{'2}, c_{ij}^{'3})$ denotes the first objective SVTrN shipping cost associated with i^{th} resource to j^{th} activity,

$\tilde{t}_{ij}^N = (t_{ij}^1, t_{ij}^2, t_{ij}^3, t_{ij}^4; t_{ij}^{'1}, t_{ij}^{'2}, t_{ij}^{'3})$ denotes the second objective SVTrN transportation time associated with i^{th} resource to j^{th} activity,

$\tilde{x}_{ij}^N = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4; x_{ij}^{'1}, x_{ij}^{'2}, x_{ij}^{'3})$ denotes the SVTrN variable assuming 0 or 1 depending upon the entire assignment of j^{th} activity fulfilled from i^{th} resource.

Using the score function [7] to the objective function of (P_1) , we get

$$(P_2) \text{ Minimize } \Re \left(Z^{(1)N}(x) \right) = \Re \left(\sum_{i=1}^n \sum_{j=1}^n c_{ij}^1 x_{ij}^1, c_{ij}^2 x_{ij}^2, c_{ij}^3 x_{ij}^3, c_{ij}^4 x_{ij}^4; c_{ij}^{'1} x_{ij}^{'1}, c_{ij}^{'2} x_{ij}^{'2}, c_{ij}^{'3} x_{ij}^{'3} \right). \quad (6)$$

$$\text{Minimize } \Re \left(Z^{(2)N}(x) \right) = \Re \left(\sum_{i=1}^n \sum_{j=1}^n d_{ij}^1 x_{ij}^1, d_{ij}^2 x_{ij}^2, d_{ij}^3 x_{ij}^3, d_{ij}^4 x_{ij}^4; d_{ij}^{'1} x_{ij}^{'1}, d_{ij}^{'2} x_{ij}^{'2}, d_{ij}^{'3} x_{ij}^{'3} \right). \quad (7)$$

Subject to:

$$\sum_{j=1}^n x_{ij}^1 = 1^N, \quad \sum_{j=1}^n x_{ij}^2 = 1^N, \quad \sum_{j=1}^n x_{ij}^3 = 1^N, \quad \sum_{j=1}^n x_{ij}^4 = 1^N, \quad i = 1, 2, \dots, n, \quad (8)$$

$$\sum_{j=1}^n x'_{ij}{}^1 = 1^N, \quad \sum_{j=1}^n x'_{ij}{}^2 = 1^N, \quad \sum_{j=1}^n x'_{ij}{}^3 = 1^N, \quad i = 1, 2, \dots, n, \quad (9)$$

$$\sum_{i=1}^n x_{ij}^1 = 1^N, \quad \sum_{i=1}^n x_{ij}^2 = 1^N, \quad \sum_{i=1}^n x_{ij}^3 = 1^N, \quad \sum_{i=1}^n x_{ij}^4 = 1^N, \quad j = 1, 2, \dots, n, \quad (10)$$

$$\sum_{i=1}^n x'_{ij}{}^1 = 1^N, \quad \sum_{i=1}^n x'_{ij}{}^2 = 1^N, \quad \sum_{i=1}^n x'_{ij}{}^3 = 1^N, \quad j = 1, 2, \dots, n, \quad (11)$$

$$\begin{aligned} x_{ij}^1 &= 0^N \text{ or } 1^N, & x_{ij}^1 - x'_{ij}{}^1 &= 0^N \text{ or } 1^N, & x'_{ij}{}^2 - x_{ij}^1 &= 0^N \text{ or } 1^N, \\ x_{ij}^2 - x'_{ij}{}^2 &= 0^N \text{ or } 1^N, & x_{ij}^3 - x'_{ij}{}^3 &= 0^N \text{ or } 1^N, & x'_{ij}{}^3 - x_{ij}^3 &= 0^N \text{ or } 1^N, \\ x_{ij}^4 - x'_{ij}{}^3 &= 0^N \text{ or } 1^N, & & \text{for all } i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, n. \end{aligned}$$

The following theorem proves that the efficient solution obtained from the problem (P_1) is equivalent to the efficient solution obtained from problem (P_2) which is based on [18].

Theorem 3.1. Let,

$$\hat{x}^N = \left\{ \left(\hat{x}_{ij}^1, \hat{x}_{ij}^2, \hat{x}_{ij}^3, \hat{x}_{ij}^4; \hat{x}'_{ij}{}^1, \hat{x}'_{ij}{}^2, \hat{x}'_{ij}{}^3 \right), \quad i = 1, 2, 3, \dots, n, \quad j = 1, 2, 3, \dots, n \right\},$$

be an efficient solution to the problem (P_2) then,

$$\bar{x}^N = \left\{ \left(\bar{x}_{ij}^1, \bar{x}_{ij}^2, \bar{x}_{ij}^3, \bar{x}_{ij}^4; \bar{x}'_{ij}{}^1, \bar{x}'_{ij}{}^2, \bar{x}'_{ij}{}^3 \right), \quad i = 1, 2, 3, \dots, n, \quad j = 1, 2, 3, \dots, n \right\},$$

is an efficient solution to the problem (P_1) .

Proof. Since $\bar{x}^N$ be an efficient solution to the problem (P_2) then $\bar{x}^N$ is a feasible solution to the problem (P_1) .

$$\begin{aligned} \sum_{j=1}^n \hat{x}_{ij}^l &= 1^N, \quad i = 1, 2, 3, \dots, n, \quad l = 1, 2, 3, 4; & \sum_{j=1}^n \hat{x}_{ij}^d &= 1^N, \quad i = 1, 2, 3, \dots, n, \quad d = 1, 2, 3, \\ \sum_{i=1}^n \hat{x}_{ij}^l &= 1^N, \quad j = 1, 2, 3, \dots, n, \quad l = 1, 2, 3, 4; & \sum_{i=1}^n \hat{x}_{ij}^d &= 1^N, \quad j = 1, 2, 3, \dots, n, \quad d = 1, 2, 3, \\ \hat{x}_{ij}^N &= 0^N \text{ or } 1^N, \quad \text{for all } i \text{ and } j. \end{aligned}$$

Assume that $\bar{x}^N$ is not an efficient solution to the problem (P_1) then there exists any point $\hat{x}^N$ such that,

$$\Re(Z^{(m)N}(\hat{x}^N)) \geq \Re(Z^{(m)N}(\bar{x}^N)) \quad \text{and} \quad \Re(Z^{(m)N}(\hat{x}^N)) > \Re(Z^{(m)N}(\bar{x}^N)),$$

for at least one m .

Using Definition 2.6, there exists any point $\hat{x}^N$ such that,

$$(Z^{(m)N}(\hat{x}^N)) \geq (Z^{(m)N}(\bar{x}^N)) \quad \text{and} \quad (Z^{(m)N}(\hat{x}^N)) > (Z^{(m)N}(\bar{x}^N)),$$

for at least one m .

Therefore, $\hat{x}^N$ is not an efficient solution to the problem (P_2) . This is contradiction. Hence, the result. $\square$

It is impossible to solve the reduced deterministic problem explicitly. Although, numerous approaches exist in the literature to reduce the multi-objective optimization problem (MOOP) into the single objective optimization problem (SOOP). The following section discusses the various proposed transformation approaches to reduce the bi-objective AP into its single objective AP.

4 Transformation Approaches

Multi-objective problems are converted into single-objective problems in situations where decision-making involves multiple conflicting goals and unified solution are needed. In resource allocation, businesses may need to simultaneously consider profitability, risk and employee satisfaction. Similarly, in scheduling and assignment problems such as assigning staff to shifts based on preferences, workload and cost need to be optimized together. These transformations are particularly useful when the DM wants a simplified decision process or when the available optimization tools are designed to handle only a single objective. Various transformation techniques such as linear, hyperbolic, exponential MF, GCM, WSM, FPA and NCPA are developed to reduce the multi-objective problem under neutrosophic environment into single objective problem under neutrosophic environment. This paper utilizes transformation techniques for deterministic BOAP, enabling us to effectually evaluate the relative efficacy of decision-making units.

1. Linear MF:

The linear MF is one of the key assumptions to tackle the optimization problems during the process of decision-making. Due to its simplicity, linear MF is commonly applied in analysing the decision, but it is used when the marginal frequency of objective function remains constant. Linear MF is suitable when satisfaction changes proportionally with the objective function. The DMs can prefer this MF due to its equal importance across each objective function.

2. Hyperbolic MF:

The hyperbolic MF have a concave part on one side and a convex part on the other side. The convex form shows decision-making behaviour in which the level of satisfaction is greater whereas, the concave component gives a lower level of satisfaction. This is suitable for the DMs who can tolerate small deviations from the optimal solution but become increasingly dissatisfied with larger deviations, reflecting a more risk-averse attitude.

3. Exponential MF:

An exponential MF occurs when the DM is worse off concerning an objective function and chooses a greater marginal frequency of satisfaction. It is suitable when optimal solutions are strongly preferred, and any deviation causes significant dissatisfaction. Thus by choosing the exponential MF, the DM can also minimize the gaps by selecting suitable shape parameters.

4. FPA:

Fuzzy programming approach (FPA) transforms the multiple fuzzy goals into a single objective by using membership function that represent the degree to which a solution satisfies each objective. In FPA, we determine the upper bound as and lower bound as for objective

function where represents the maximum level of performance that is deemed acceptable for the m th objective and is the minimum level of performance that is considered satisfactory for the m th objective. This approach allows the DMs to specify their preferences and priorities in terms of what levels of performance are acceptable or desirable for each criterion.

5. NCPA:

Neutrosophic compromise programming approach (NCPA) extends fuzzy programming by incorporating indeterminacy and inconsistency which are common in real-world decision-making. The NCPA is an optimization technique based on neutrosophic set theory to handle decision-making problems. It aims by maximizing the truth and indeterminacy and by minimizing the falsity to find OCS. This approach assists a flexible framework for DM to present their neutral views and manage uncertainties efficiently.

6. GCM:

The global criteria method (GCM) transforms the multi-objective optimization into single objective optimization by minimizing the global distance from the ideal solution. Initially, it is necessary to determine the optimal solution for all the objective function separately. The objective function is formulated to minimize the difference between objective function and optimal solution. This approach is particularly useful when the goal is to treat all objectives equally without assigning subjective weights.

7. WSM:

The weighted sum method (WSM) transforms the multi-objective problem into single-objective problem by assigning weights to each objective function to form a single objective. The objective is to maximize or minimize the weighted sum based on the DMs preferences. DM assigns weights according to the scale of the objective function. While evaluating these objective functions, it is preferable to scale them such that all the objective function have the same magnitude. It is useful when the relative importance of objectives is known and can be quantified through weights.

5 Solution Approach

In this section, we discuss the various transformation techniques such as linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM. These transformation techniques are incorporated based on a neutrosophic environment which consists of maximizing truth and indeterminacy and by minimizing falsity. It helps the DMs to decide their neutral thought and to overcome these techniques in an efficient manner. Figure 1 represents the flowchart of the solution approach. These techniques are employed to reduce the MOP into SOP which is proceeds as follows:

Step 1: Consider NBOAP (P_1).

Step 2: Transform the NBOAP (P_1) into its equivalent deterministic BOAP (P_2) using the score function [7].

Step 3: Check whether the reduced deterministic BOAP is balanced. If not, balance it and then solve $Z^{(1)}(x)$ and $Z^{(2)}(x)$ of deterministic BOAP to obtain the optimal solution using Hungarian method (HM). Then using the obtained optimal solution, the pay-off matrix is constructed in Table 2.

Table 2: Pay-off matrix.

	$Z^{(1)}$	$Z^{(2)}$
X_1	$Z^{(1)}(X_1)$	$Z^{(2)}(X_1)$
X_2	$Z^{(1)}(X_2)$	$Z^{(2)}(X_2)$

Step 4: Calculate the upper bound (UB) U_m and lower bound (LB) L_m for the problem (B) under neutrosophic environment are as follows;

$$U_m = \max \left[Z^{(m)}(X_m) \right], \tag{12}$$

$$L_m = \min \left[Z^{(m)}(X_m) \right]. \tag{13}$$

Step 5: Calculate the UB and LB for truth, indeterminacy and falsity degree under neutrosophic environment from Table 1 using the equations provided by [2] which are given as follows;

$$U_m^\mu = U_m, \qquad L_m^\mu = L_m \text{ for truth degree,} \tag{14}$$

$$U_m^\eta = L_m^\mu + s_m, \qquad L_m^\eta = L_m^\mu \text{ for indeterminacy degree,} \tag{15}$$

$$U_m^\nu = U_m^\mu, \qquad L_m^\nu = L_m^\mu + t_m \text{ for falsity degree.} \tag{16}$$

Step 6: Transform the bi-objective AP into its single objective AP using the various proposed transformation approaches such as linear MF [2], hyperbolic MF [2], exponential MF [2], GCM [20], WSM [32], FPA [34] and NCPA [1] under neutrosophic environment according to the preference of DM.

Step 7: Solve the reduced problem to obtain the OCS using LINGO software. The obtained optimal allotments of the problem (P_2) are used to find the neutrosophic OCS of the problem (P_1) . Figure 1 represents the flowchart for the solution approach.

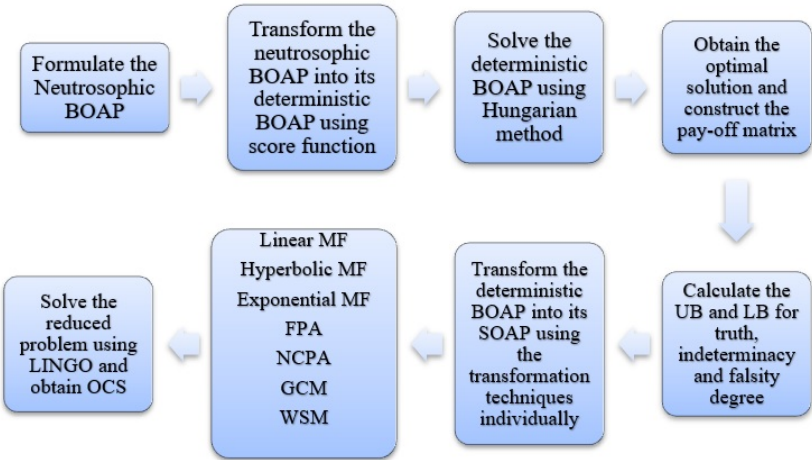


Figure 1: Flowchart of the solution approach.

The solution approach for solving the neutrosophic BOAP is demonstrated by numerical examples which are given in the following section. In the below numerical examples, we transform the NBOAP into its equivalent deterministic BOAP using score

function. The reduced deterministic BOAP is extremely difficult and consumes more time to obtain an optimal solution. Due to this reason, the reduced deterministic BOAP cannot be solved explicitly. So, we convert the deterministic BOAP into its SOAP using various transformation techniques. The reduced problem is then solved by LINGO to find OCS.

6 Numerical Illustration

Example 6.1. *The hospital intends to purchase three distinct types of medical devices such as Anesthesia machines (D_1), Respiratory ventilators (D_2) and Defibrillators (D_3). The three different manufacturers such as Abbot laboratories (M_1), Medtronic (M_2) and Johnson and Johnson (M_3) have expressed interest in supplying one or all of the devices. Abbot laboratories provides good quality anesthesia machines and offers a return policy. Similarly, Respiratory ventilators and defibrillators are good quality products with return policies from Medtronic and Johnson and Johnson, respectively. So, the hospitals policy prohibits purchasing more than one device from a single manufacturer at a time. The goal of the hospital is to reduce the shipping cost and deterioration cost by assigning the three different manufacturers to the three distinct types of medical devices. In these situations, the parameters such as shipping cost and deterioration cost are imprecise due to some unpredictable reasons such as opinion by the doctors and fluctuations in market. Sometimes these type of uncertain data is stored in terms of truth, indeterminacy and falsity degree. Under this circumstances, the shipping cost and deterioration cost arise in the structure of SVTrNNs. Table 3 shows the data of the assignment cost and assignment time.*

Table 3: SVTrNBOAP.

		D_1	D_2	D_3
M_1	$\tilde{Z}^{(1)N}$	$\tilde{c}_{11}^N$	$\tilde{c}_{12}^N$	$\tilde{c}_{13}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{11}^N$	$\tilde{t}_{12}^N$	$\tilde{t}_{13}^N$
M_2	$\tilde{Z}^{(1)N}$	$\tilde{c}_{21}^N$	$\tilde{c}_{22}^N$	$\tilde{c}_{23}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{21}^N$	$\tilde{t}_{22}^N$	$\tilde{t}_{23}^N$
M_3	$\tilde{Z}^{(1)N}$	$\tilde{c}_{31}^N$	$\tilde{c}_{32}^N$	$\tilde{c}_{33}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{31}^N$	$\tilde{t}_{32}^N$	$\tilde{t}_{33}^N$

where

$$\begin{aligned}\tilde{c}_{11}^N &= (14, 17, 21, 28; 0.8, 0.2, 0.6), \\ \tilde{c}_{12}^N &= (13, 18, 20, 24; 0.6, 0.4, 0.5), \\ \tilde{c}_{13}^N &= (20, 25, 30, 35; 0.8, 0.4, 0.2), \\ \tilde{c}_{21}^N &= (15, 18, 23, 30; 0.9, 0.2, 0.3), \\ \tilde{c}_{22}^N &= (11, 16, 25, 28; 0.8, 0.3, 0.2), \\ \tilde{c}_{23}^N &= (17, 19, 23, 27; 0.9, 0.3, 0.2), \\ \tilde{c}_{31}^N &= (11, 17, 22, 25; 0.6, 0.5, 0.4), \\ \tilde{c}_{32}^N &= (10, 14, 26, 30; 0.8, 0.6, 0.2), \\ \tilde{c}_{33}^N &= (14, 16, 21, 23; 0.7, 0.5, 0.3),\end{aligned}$$

$$\begin{aligned}\tilde{t}_{11}^N &= (12, 20, 25, 29; 0.9, 0.3, 0.2), \\ \tilde{t}_{12}^N &= (22, 25, 30, 34; 0.8, 0.2, 0.4), \\ \tilde{t}_{13}^N &= (12, 18, 21, 24; 0.7, 0.4, 0.5), \\ \tilde{t}_{21}^N &= (15, 17, 19, 24; 0.7, 0.2, 0.3), \\ \tilde{t}_{22}^N &= (28, 32, 35, 40; 0.9, 0.3, 0.2), \\ \tilde{t}_{23}^N &= (12, 15, 27, 29; 0.8, 0.2, 0.3), \\ \tilde{t}_{31}^N &= (23, 27, 30, 31; 0.9, 0.3, 0.4), \\ \tilde{t}_{32}^N &= (12, 19, 24, 25; 0.8, 0.5, 0.4), \\ \tilde{t}_{33}^N &= (13, 18, 23, 25; 0.9, 0.2, 0.2).\end{aligned}$$

Solving the values of Table 3 directly will consume more time and more effort. As well as sometimes, Table 3 cannot be solved explicitly since it provides the negative value for the decision variable which violates the non-negativity constraints of the optimization problem. Due to these reasons, we have applied score function to convert the neutrosophic numbers into their equivalent deterministic numbers. Using Definition 2.5, the deterministic value of score function for the first cell from Table 3. Let us consider the first cell as;

$$\tilde{c}_{11}^N = (14, 17, 21, 28; 0.8, 0.2, 0.6) ,$$
$$S\left(\tilde{c}_{11}^N\right)=\frac{1}{16}\left[14+17+21+28\right](2+0.8-0.2-0.6)=10.$$

In the same manner, we compute the deterministic values using score function of the other cells in Table 3. The reduced deterministic BOAP is shown in Table 4.

Table 4: Deterministic BOAP.

		D_1	D_2	D_3
M_1	$Z^{(1)}$	10	8	15
	$Z^{(2)}$	13	15	8
M_2	$Z^{(1)}$	13	12	13
	$Z^{(2)}$	10	20	12
M_3	$Z^{(1)}$	8	10	9
	$Z^{(2)}$	15	10	12

As in Step 3, the above deterministic BOAP is balanced. Using HM, the optimal allotment of $Z^{(1)}(x)$ is $M_1 \rightarrow D_2, M_2 \rightarrow D_3, M_3 \rightarrow D_1$ and the optimal solution is 29. Similarly, the optimal allotment of $Z^{(2)}(x)$ is $M_1 \rightarrow D_3, M_2 \rightarrow D_1, M_3 \rightarrow D_2$ and the optimal solution is 28. Using the obtained optimal solutions of $Z^{(1)}(x)$ and $Z^{(2)}(x)$, the pay-off matrix is constructed in Table 5.

Table 5: Pay-off matrix.

	$Z^{(1)}$	$Z^{(2)}$
X_1	29	42
X_2	38	28

Using Step 4, the UB and LB for the problem (P_2) using (12) and (13) are calculated as $U_1 = 38, L_1 = 29, U_2 = 42$ and $L_2 = 28$.

As in Step 5, the UB and LB for truth, indeterminacy and falsity MF under neutrosophic environment using (14) to (16) are calculated as follows:

- The truth MF using (14) is $U_1^\mu = 38, L_1^\mu = 29, U_2^\mu = 42$ and $L_2^\mu = 28$.
- The indeterminacy MF using (15) is $U_1^\eta = 29 + s_1, L_1^\eta = 29, U_2^\eta = 28 + s_2$ and $L_2^\eta = 28$.
- The falsity MF using (16) is $U_1^\nu = 38, L_1^\nu = 29 + t_1, U_2^\nu = 42$ and $L_2^\nu = 28 + t_2$.

The reduced deterministic BOAP has two conflicting objective functions which tend to achieve optimum values. It is extremely difficult and consumes more time to obtain the optimal solution. Due to this reason, the researchers introduced various techniques to transform the MOOP into

the SOOP. Researchers such as Zadeh [32] introduced the weighted sum method to reduce the MOLPP into LPP, Zimmermann [34] introduced the fuzzy programming approach to reduce the MOLPP into LPP [20] introduced the global criteria method to reduce the fuzzy MOTP into fuzzy TP, Ahmad and Adhami [1] introduced the neutrosophic compromise programming approach to reduce the fuzzy MOTP into fuzzy TP and Ahmadini and Ahmad [2] introduced the linear, hyperbolic and exponential MF to reduce the intuitionistic MOLPP into intuitionistic LPP. In this article, to reduce the deterministic BOAP into its SOAP, we employ transformation techniques available in the literature such as linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM. The reduced problem is then solved by LINGO to determine OCS which is summarized in Table 6.

Table 6: Problem formulation and solutions of the transformation approaches.

Transformation approaches	Problem formulation	Optimal allotment	Optimal compromise solution
Linear MF	$\max \alpha - \beta - \gamma$ Subject to: $Z^{(1)}(x) + 9\alpha \leq 38,$ $Z^{(2)}(x) + 14\alpha \leq 42,$ $Z^{(1)}(x) - s_1\beta \leq 29,$ $Z^{(2)}(x) - s_2\beta \leq 28,$ $Z^{(1)}(x) - 9\gamma + t_1\gamma \leq 29 + t_1,$ $Z^{(2)}(x) - 14\gamma + t_2\gamma \leq 28 + t_2,$ where $\alpha \geq \beta, \alpha \geq \gamma, \alpha + \beta + \gamma \leq 3,$ $\alpha, \beta, \gamma \in (0, 1).$ $\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3.$ $\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3.$ $x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$	$M_1 \rightarrow D_1,$ $M_2 \rightarrow D_3,$ $M_3 \rightarrow D_2.$	(33, 35)
Neutrosophic optimal compromise solution		$\{(40, 46, 6, 84; 0.8, 0.6, 0.6),$ $(41, 57, 71, 80; 0.8, 0.5, 0.4)\}$	
Hyperbolic MF	$\max \mu - \eta - \nu$ Subject to: $0.11 [Z^{(1)}(x)] + \mu \leq \frac{0.11}{2}(38 + 29),$ $0.428 [Z^{(2)}(x)] + \mu \leq \frac{0.428}{2}(42 + 28),$ $0.11 [Z^{(1)}(x)] - \eta \leq \frac{0.11}{2}(29 + s_1 + 29),$ $0.428 [Z^{(2)}(x)] - \eta \leq \frac{0.428}{2}(28 + s_2 + 28),$ $0.11 [Z^{(1)}(x)] - \nu \leq \frac{0.11}{2}(38 + 29 + t_1),$ $0.428 [Z^{(2)}(x)] - \nu \leq \frac{0.428}{2}(42 + 28 + t_2),$ where $\mu \geq \eta, \mu \geq \nu, \mu + \eta + \nu \leq 3,$ $\mu, \eta, \nu \in (0, 1).$	$M_1 \rightarrow D_1,$ $M_2 \rightarrow D_3,$ $M_3 \rightarrow D_2.$	(33, 35)

$$\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3.$$

$$\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3.$$

$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j,$$

$$\text{where } \mu = \tanh^{-1}(2\alpha - 1), \eta = \tanh^{-1}(2\beta - 1)$$

$$\text{and } \nu = \tanh^{-1}(2\gamma - 1).$$

Neutrosophic optimal compromise solution		$\{(40, 46, 6, 84; 0.8, 0.6, 0.6), (41, 57, 71, 80; 0.8, 0.5, 0.4)\}$
Exponential MF	$\max \alpha - \beta - \gamma$ Subject to: $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{9} \right) (Z^{(1)}(x) - 29) \right] - e^{-0.5} \geq \alpha,$ $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{14} \right) (Z^{(2)}(x) - 28) \right] - e^{-0.5} \geq \alpha,$ $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{s_1} \right) (29 + s_1 - Z^{(1)}(x)) \right] - e^{-0.5} \leq \beta,$ $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{s_2} \right) (28 + s_2 - Z^{(2)}(x)) \right] - e^{-0.5} \leq \beta,$ $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{9 - t_1} \right) (38 - Z^{(1)}(x)) \right] - e^{-0.5} \leq \gamma,$ $\frac{1}{1 - e^{-0.5}} \exp \left[\left(\frac{-0.5}{14 - t_2} \right) (42 - Z^{(2)}(x)) \right] - e^{-0.5} \leq \gamma,$ where $\alpha \geq \beta, \quad \alpha \geq \gamma, \quad \alpha + \beta + \gamma \leq 3;$ $\alpha, \beta, \gamma \in (0, 1).$	$M_1 \rightarrow D_1, \quad (33, 35)$ $M_2 \rightarrow D_3,$ $M_3 \rightarrow D_2$
Neutrosophic optimal compromise solution		$\{(40, 46, 6, 84; 0.8, 0.6, 0.6), (41, 57, 71, 80; 0.8, 0.5, 0.4)\}$
GCM	$\min Z(x) = \left\{ \left[\frac{Z^{(1)}(x) - 29}{38 - 29} \right]^2 + \left[\frac{Z^{(2)}(x) - 28}{42 - 28} \right]^2 \right\}^{\frac{1}{2}},$ Subject to: $\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3,$ $\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3,$ $x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$	$M_1 \rightarrow D_3, \quad (38, 28)$ $M_2 \rightarrow D_1,$ $M_3 \rightarrow D_2$
Neutrosophic optimal compromise solution		$\{(47, 57, 77, 95; 0.8, 0.6, 0.3), (39, 54, 64, 73; 0.7, 0.5, 0.5)\}$
WSM	$\min Z(x) = w_1 Z^{(1)}(x) + w_2 Z^{(2)}(x)$ Subject to:	$M_1 \rightarrow D_3, \quad (38, 28)$ $M_2 \rightarrow D_1,$

	$\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3,$ $\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3,$ $x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j,$ $\text{where } w_1 = w_2 = 0.5.$	$M_3 \rightarrow D_2.$
Neutrosophic optimal compromise solution		$\{(47, 57, 77, 95; 0.8, 0.6, 0.3), (39, 54, 64, 73; 0.7, 0.5, 0.5)\}$
FPA	$\max \mu$ Subject to: $Z^{(1)}(x) + (38 - 29)\mu \leq 38,$ $Z^{(2)}(x) + (42 - 28)\mu \leq 42,$ $\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3,$ $\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3,$ $x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$	$M_1 \rightarrow D_1, \quad (33, 35)$ $M_2 \rightarrow D_3,$ $M_3 \rightarrow D_2.$
Neutrosophic optimal compromise solution		$\{(40, 46, 6, 84; 0.8, 0.6, 0.6), (41, 57, 71, 80; 0.8, 0.5, 0.4)\}$
NCPA	$\max \alpha - \beta + \gamma$ Subject to: $Z^{(1)}(x) + 9\alpha \leq 38,$ $Z^{(2)}(x) + 14\alpha \leq 42,$ $Z^{(1)}(x) + 9s_1\beta \leq 29 + 9s_1,$ $Z^{(2)}(x) + 14s_2\beta \leq 28 + 14s_2,$ $Z^{(1)}(x) - 9\gamma + 9t_1\gamma \leq 29 + 9t_1,$ $Z^{(2)}(x) - 14\gamma + 14t_2\gamma \leq 28 + 14t_2,$ where $\alpha \geq \beta, \quad \alpha \geq \gamma, \quad \alpha + \beta + \gamma \leq 3;$ $\alpha, \beta, \gamma \in (0, 1).$ $\sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3,$ $\sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3,$ $x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$	$M_1 \rightarrow D_1, \quad (33, 35)$ $M_2 \rightarrow D_3,$ $M_3 \rightarrow D_2.$
Neutrosophic optimal compromise solution		$\{(40, 46, 6, 84; 0.8, 0.6, 0.6), (41, 57, 71, 80; 0.8, 0.5, 0.4)\}$

Finally, the corresponding neutrosophic OCS obtained for linear, hyperbolic, exponential MF, FPA and NCPA is $\{(41, 50, 70, 85; 0.8, 0.6, 0.6), (36, 54, 76, 83; 0.8, 0.5, 0.4)\}$ and the obtained neutrosophic OCS for GCM and WSM is $\{(45, 57, 79, 95; 0.8, 0.6, 0.3), (39, 54, 64, 73; 0.7, 0.5, 0.5)\}$. For better understanding, the obtained OCS compared among the transformation techniques is plotted as a bar graph using Excel as shown in Figure 2.

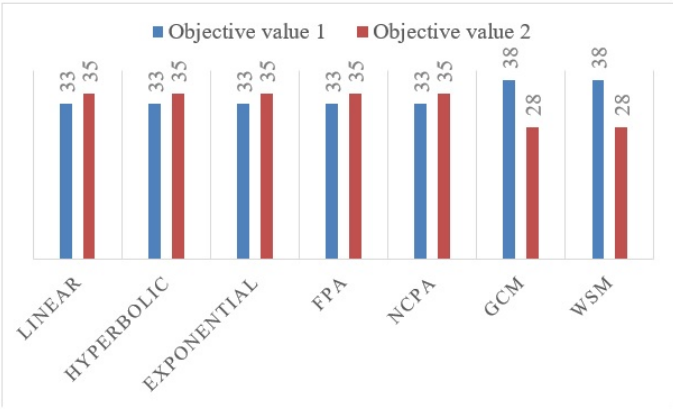


Figure 2: Comparison of transformation techniques for Example 6.1.

From Table 6 and Figure 2, it can be seen that the obtained optimal compromise solution extracted from GCM and WSM is more preferable to linear, hyperbolic, exponential MF and FPA. When the hospital seeks to maximize the profit, the DM can adopt the global criteria method to find the best solution closest to the ideal. If hospitals plans to assign nurses to shifts based on cost, preferences and workload then the WSM helps by assigning weights to each objective based on hospital priorities. When the goal of hospital is to minimize the uncertain cost and time, influenced by factors like opinions and market shifts, FPA helps the DM to manage this imprecision effectively. In addition to this, the marginal evaluation of each objective can be performed using linear, hyperbolic and exponential MF which allows the DMs to choose the most suitable MF. As per our study, the DM can adopt any transformation techniques independently which fits the problem with higher priority.

Example 6.2. The airline company has redesigned the flight schedule. The purpose of this problem is to allocate three different pilots P_1 , P_2 and P_3 to the three distinct types of flights such as Air India (F_1), IndiGo (F_2) and SpiceJet (F_3). It may not be suitable for some pilots to take some flights due to domestic restrictions. So, the policy of an airline company prohibits assigning more than one fight to a single pilot at a time. The goal of the airline company is to minimize the assignment cost and assignment time by assigning the three different pilots to the three distinct types of flights. In these situations, the parameters such as assignment cost and assignment time are imprecise due to some unpredictable reasons such as preference by the pilots and fluctuations in an airline company. Sometimes these type of uncertain data is stored in truth, indeterminacy and falsity. Under this circumstances, the assignment cost and time arise in the structure of SVTrNNs. Table 7 shows the data of the assignment cost and assignment time.

Table 7: SVTrNBOAP.

		F_1	F_2	F_3
P_1	$\tilde{Z}^{(1)N}$	$\tilde{c}_{11}^N$	$\tilde{c}_{12}^N$	$\tilde{c}_{13}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{11}^N$	$\tilde{t}_{12}^N$	$\tilde{t}_{13}^N$
P_2	$\tilde{Z}^{(1)N}$	$\tilde{c}_{21}^N$	$\tilde{c}_{22}^N$	$\tilde{c}_{23}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{21}^N$	$\tilde{t}_{22}^N$	$\tilde{t}_{23}^N$
P_3	$\tilde{Z}^{(1)N}$	$\tilde{c}_{31}^N$	$\tilde{c}_{32}^N$	$\tilde{c}_{33}^N$
	$\tilde{Z}^{(2)N}$	$\tilde{t}_{31}^N$	$\tilde{t}_{32}^N$	$\tilde{t}_{33}^N$

where

$$\begin{aligned}\tilde{c}_{11}^N &= (17, 19, 20, 22; 0.9, 0.2, 0.1), & \tilde{t}_{11}^N &= (18, 20, 21, 23; 0.9, 0.2, 0.1), \\ \tilde{c}_{12}^N &= (11, 13, 15, 16; 0.8, 0.3, 0.2), & \tilde{t}_{12}^N &= (21, 23, 25, 27; 0.7, 0.1, 0.1), \\ \tilde{c}_{13}^N &= (22, 23, 25, 26; 0.9, 0.1, 0.1), & \tilde{t}_{13}^N &= (25, 28, 30, 32; 0.9, 0.2, 0.2), \\ \tilde{c}_{21}^N &= (24, 27, 30, 32; 0.7, 0.1, 0.1), & \tilde{t}_{21}^N &= (14, 15, 17, 18; 0.7, 0.2, 0.1), \\ \tilde{c}_{22}^N &= (25, 30, 35, 38; 0.8, 0.2, 0.2), & \tilde{t}_{22}^N &= (28, 32, 35, 40; 0.9, 0.3, 0.2), \\ \tilde{c}_{23}^N &= (12, 14, 16, 18; 0.9, 0.1, 0.3), & \tilde{t}_{23}^N &= (16, 17, 19, 20; 0.9, 0.1, 0.1), \\ \tilde{c}_{31}^N &= (20, 21, 23, 25; 0.9, 0.1, 0.1), & \tilde{t}_{31}^N &= (20, 22, 23, 25; 0.9, 0.2, 0.1), \\ \tilde{c}_{32}^N &= (32, 35, 38, 40; 0.8, 0.1, 0.1), & \tilde{t}_{32}^N &= (15, 16, 18, 19; 0.8, 0.1, 0.3), \\ \tilde{c}_{33}^N &= (12, 14, 15, 17; 0.9, 0.3, 0.2), & \tilde{t}_{33}^N &= (18, 20, 21, 23; 0.7, 0.1, 0.3).\end{aligned}$$

Using Steps 1 to 6, the obtained optimal allotment for linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM is $P_1 \rightarrow F_2, P_2 \rightarrow F_1$ and $P_3 \rightarrow F_3$ and OCS is (35, 37). Finally, the corresponding neutrosophic OCS for linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM is $\{(47, 54, 60, 65; 0.7, 0.3, 0.2), (53, 58, 63, 68; 0.7, 0.2, 0.3)\}$. For better understanding, the obtained OCS compared among the transformation techniques is plotted as a bar graph using Excel as shown in Figure 3.

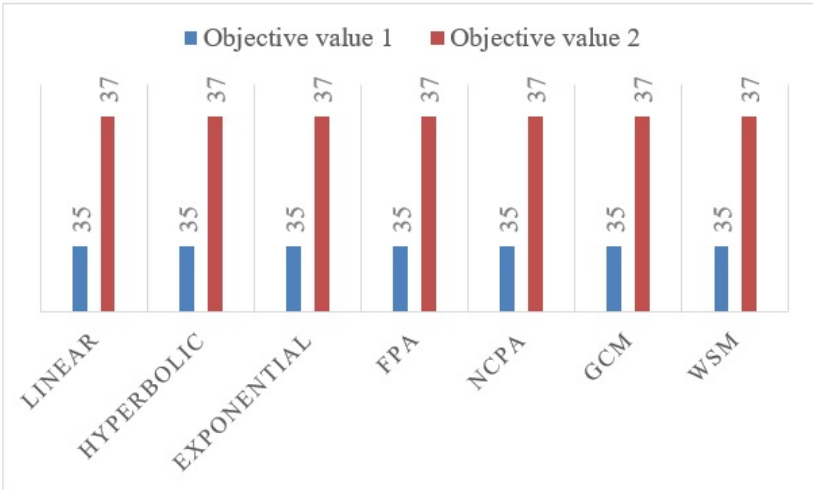


Figure 3: Comparison of transformation techniques for Example 6.2.

From Figure 3, it is analysed that the obtained OCS extracted from the linear, hyperbolic, exponential MF, FPA, NCPA, GCM and WSM provides the same result. As per our study, the DM can adopt any transformation techniques independently which fits the problem with higher priority.

7 Sensitivity Analysis

The sensitivity analysis involves altering the parameters of objective function by increasing or decreasing a certain percentage of values, while holding the other parameters at their initial values. First we alter the parameters of objective function by decreasing the value as 5% using linear MF. Then the linear MF is formulated as;

$$\max \alpha - \beta - \gamma,$$

Subject to:

$$\begin{aligned} 9.5x_{11} + 7.6x_{12} + 14.25x_{13} + 12.35x_{21} + 11.4x_{22} + 12.35x_{23} + 7.6x_{31} + 9.5x_{32} + 8.55x_{33} + 9\alpha &\leq 38, \\ 12.35x_{11} + 14.25x_{12} + 7.6x_{13} + 9.5x_{21} + 19x_{22} + 11.4x_{23} + 14.25x_{31} + 9.5x_{32} + 11.4x_{33} + 14\alpha &\leq 42, \\ 9.5x_{11} + 7.6x_{12} + 14.25x_{13} + 12.35x_{21} + 11.4x_{22} + 12.35x_{23} + 7.6x_{31} + 9.5x_{32} + 8.55x_{33} - s_1\beta &\leq 29, \\ 12.35x_{11} + 14.25x_{12} + 7.6x_{13} + 9.5x_{21} + 19x_{22} + 11.4x_{23} + 14.25x_{31} + 9.5x_{32} + 11.4x_{33} - s_2\beta &\leq 28, \\ 9.5x_{11} + 7.6x_{12} + 14.25x_{13} + 12.35x_{21} + 11.4x_{22} + 12.35x_{23} + 7.6x_{31} + 9.5x_{32} + 8.55x_{33} - 9\gamma + t_1\gamma &\leq 29 + t_1, \\ 12.35x_{11} + 14.25x_{12} + 7.6x_{13} + 9.5x_{21} + 19x_{22} + 11.4x_{23} + 14.25x_{31} + 9.5x_{32} + 11.4x_{33} - 14\gamma + t_2\gamma &\leq 28 + t_2, \end{aligned}$$

$$\begin{aligned} \alpha \geq \beta, \quad \alpha \geq \gamma, \quad \alpha + \beta + \gamma \leq 3, \quad \alpha, \beta, \gamma \in (0, 1), \\ \sum_{j=1}^3 x_{ij} = 1, \quad i = 1, 2, 3, \\ \sum_{i=1}^3 x_{ij} = 1, \quad j = 1, 2, 3, \\ x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j. \end{aligned}$$

In the same manner, we have altered the other parameters of objective functions by increasing or decreasing a certain percentage of values, while holding the other parameters at their initial values for Examples 6.1 and 6.2 which are shown in Table 8.

Table 8: Variations of objective function parameter for Examples 6.1 and 6.2.

Percentage values	Example 6.1		Example 6.2	
	Z_1	Z_2	Z_1	Z_2
−5%	31.35	33.25	33.23	35.14
−3%	32.01	33.95	33.95	35.88
−1%	32.67	34.65	34.65	36.62
0	33.00	35.00	35.00	37.00
+1%	33.33	35.35	35.35	37.37
+3%	33.99	36.05	36.05	38.11
+5%	34.65	36.75	36.75	38.85

For better understanding, using Table 8 we have represented the graphical representation using MATLAB in Figures 4 and 5.

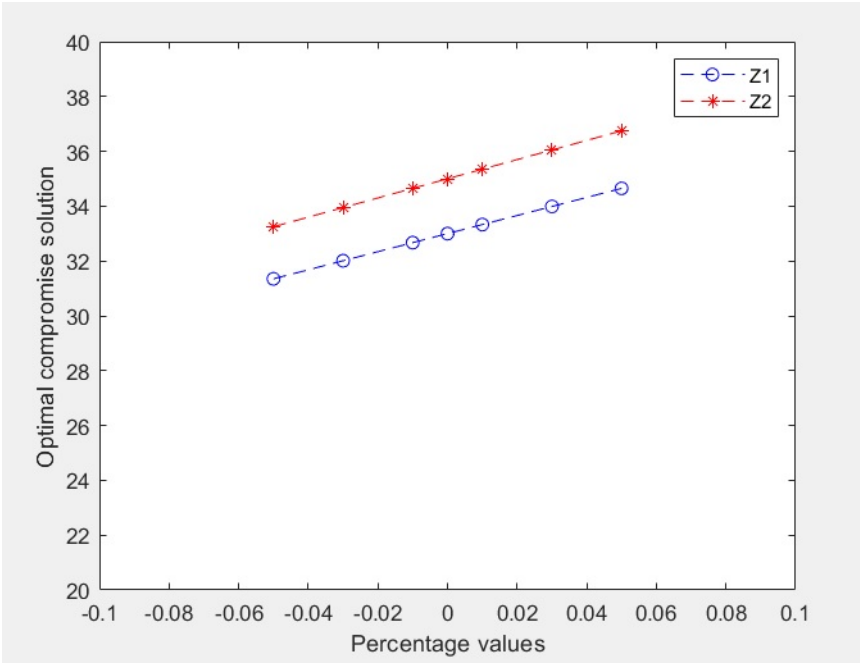


Figure 4: Sensitivity analysis for Example 6.1.

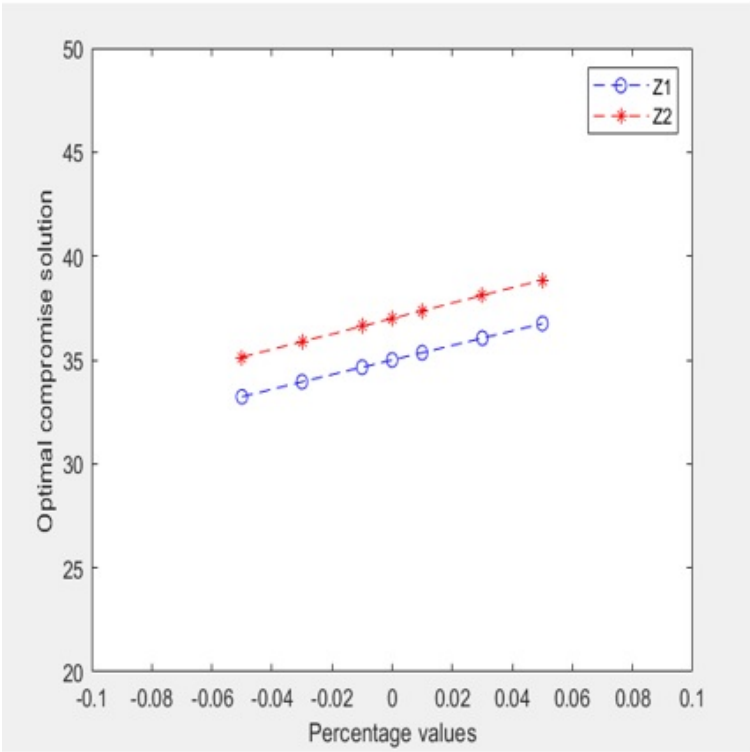


Figure 5: Sensitivity analysis for Example 6.2.

From Figures 4 and 5, we conclude that the OCS for both the objective functions gradually increases as the percentage values rise, depending on the variations in the objective function parameter.

8 Conclusions

The study considers the bi-objective assignment problem under neutrosophic environment. The neutrosophic problem is converted into its deterministic problem using score function. The key contribution of this study are summarized as:

- The key findings of this study is to develop various transformation techniques for AP to transform the multi-objective problem under neutrosophic environment into single objective problem under neutrosophic environment in order to obtain the optimal compromise solution. This paper utilizes transformation techniques for deterministic BOAP, enabling us to effectually evaluate the efficacy in decision-making situations.
- To highlight the practical efficacy of the problem, we provide two numerical examples that validates its effectiveness and significance for DM. The obtained optimal compromise solution extracted from GCM and WSM is more preferable to linear, hyperbolic, exponential MF, FPA and NCPA. Finally, we have conducted the sensitivity analysis for the objective functions.

- These transformation techniques can be applied across a variety of organizations to assist the DM in making informed decisions and optimize resource allocation which is especially useful in today's highly competitive business world.

For the future perspective, we will propose a new algorithm for this model and generalize this concept over the multi-objective fractional AP under an uncertain environment. Additionally, the techniques discussed in this study are applicable to handle a variety of real-life scenarios.

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